A New Look at Snell's Law

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Let us consider a charge packet σ(x,t) moving along the x-axis at constant speed v. (fig.1)

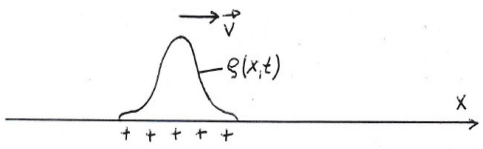

figure 1

Hence

(1) $\sigma(x,t) = \sigma(x-vt)$

We only look at a 2-dimensional situation. The charge is then a strip extending into the z-axis.
Let us also suppose that the charge be concentrated like a surface charge at y = 0.
Fourier transformation of σ(x,t) establishes a link between wavevector k_x and frequency ω

(2) $\omega = vk_x$

The current is obtained using the continuity equation in one dimension and constant velocity v

(3) $\dot{\sigma} = -v\partial_x \sigma = -\partial_x j(x,t)$

where we identify

(4) $v\sigma = j(x,t)$

In order to find the electromagnetic potentials, we treat the charge as a *boundary condition*.
See figure 2 along with the text.

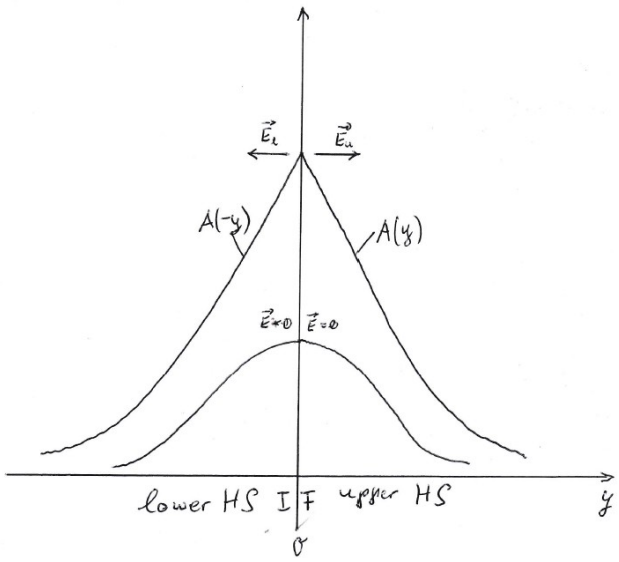

fig. 2

A solution in the upper half-space (HS) is symmetrically replicated in the lower HS.
At the interface (IF), potential slope (E_y-field) is equal and opposite on either side.
E_y-difference at IF gives the surface charge at y = 0.
For zero local charge, potential slope is zero at y = 0 (lower curve)

(5) $$2E_y(x,t,y=0)=\sigma(x,t)$$
$$E_y(x,t,y=0)=\frac{1}{2}\sigma(x,t)$$

Fouriertransformation on either side gives

(6) $$E_y(k_x,\omega,y=0)=\frac{1}{2}\sigma(k_x,\omega)$$

This enforces the relation (2) between frequency and wavevector also for the potentials.

All vectorpotentials (f) obey the free wave equation (Lorentz gauge)

(7) $$[\omega^2-c^2k_x^2]f(\omega,k_x,y)=-c^2\partial_y^2 f(\omega,k_x,y)$$

Inserting (2) gives

(8) $$[v^2-c^2]k_x^2 f(\omega,k_x,y)=-c^2\partial_y^2 f(\omega,k_x,y)$$

We have now two cases:

- for *subluminal* velocities v < c, $[v^2-c^2]<0$ hence

(9) $$[c^2-v^2]k_x^2 f(\omega, k_x, y)=c^2\partial_y^2 f(\omega, k_x, y)$$

The solution in the y-coordinate is then an *exponentially falling* function

(10) $$f(k_x, y)\propto e^{-k_y y}$$

with

(11) $$k_y=(\sqrt{1-\frac{v^2}{c^2}})k_x$$

This corresponds to *electrostatic* fields of moving charges

- for superluminal velocities v > c, $[v^2-c^2]>0$

and the solution in the y-coordinate is then an *oscillating* function

(12) $$f(k_x, y)\propto e^{ik_y y}$$

with

(13) $$k_y=(\sqrt{\frac{v^2}{c^2}-1})k_x$$

This corresponds to *radiative* fields of moving charges

Connection to surface currents is done through the B-field

(14) $$B_z(x, y=0, t)=\frac{1}{2}j(x, t)$$

It becomes clear that the rather static charge package is filled with plenty of oscillations that superpose.
Current oscillations produce radiation and the fact that we do not get any at subluminal speeds is that the interference pattern created by each Fouriercomponent only allows for evanescent waves.

Transverse waves give the same result.
We connect to transverse surface currents through

(15) $$B_x(x, y=0, t)=\frac{1}{2}j_z(x, t)$$

This situation of a charge package moving along at any speed can be simulated by the interface between a dielectric medium and vacuum (n-to-1interface), on which a plane light pulse impinges from the dielectric side with the indicated orientation of its electric field. (fig. 3)

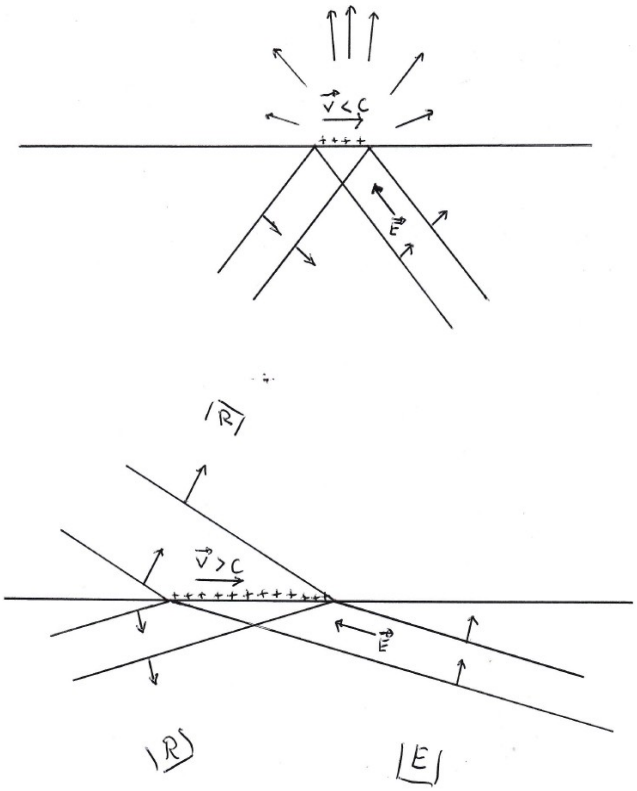

fig. 3

There will be two cases, derived from Snell's law:

- total reflection, which is tantamount to a *subluminally* moving charge package
- transmission/partial reflection for a *superluminally* moving charge package.

For total reflection, one just gets the electro/magneto-static fields of a moving charge satisfying essentially Gauss' law.
For transmission, the transmitted light can be interpreted as emanating from the superluminally moving charge, with the angle of emission being the *Mach* angle given by the ratio v/c.
The same applies to the reflected wave. It, too, is light generated by the superluminal charge, emanating under a, steeper, Mach angle into the dielectric medium.
The interface situation of figure 3 tells us more: the incident wavefront exerts a *longitudinal* force on the surface current which feeds energy in.
At superluminal speeds, this energy is radiated as reflected and transmitted (r/t) radiation;
in turn, r/t radiation exert l*ongitudinal* forces on the surface charge pulling energy out and compensating the energy inflow.
At subluminal speeds, when we get total reflection, the forces on the vacuum side are all *normal* to the velocity, hence exert no longitudinal action, neither accelerating nor braking.
The analogy of a moving charge with the n-t-1 interface brings out the interference effect already mentioned.
At subluminal speeds, the wavefront hits a "grating" with too narrow slits and only evanescent waves are possible.

Cerenkov effect

It is well known that charges injected into a dielectric (gas) whose speed exceeds the local speed of light (c/n), will emit radiation under the Mach angle.
The question remains, however, whether the energy of this radiation is generated just upon the entrance of the particles into the gas, or whether it is siphoned off the particle's kinetic energy continuously.
We see from the foregoing that the latter is the case: there is no need for material inhomogeneity, nor does the particle recover any energy upon leaving the gas and resuming its subluminal electrostatic field.

Notice that, although no overall acceleration of the charge package is involved here, to put it poetically, "the wave equation's eyes" can resolve the package's Fouriercontent and the oscillations, hence accelerations contained.
We usually have dispersion (n(ω)) and the gas will radiate only in a certain frequency range.
The color of Cerenkov radiation will depend on gas used.

Accelerated motion

When the n-to-1 interface has a spatially varying n(x), the charge package will move at varying speeds. We get overall linear "acceleration"
Accelerated currents radiate and, even in a situation of total reflection, there should be bremsstrahlung leaking out normal to the surface.
Can we understand this linear bremsstrahlung in terms of our Fourier-Interference picture?
Yes, we can: in accelerated motion, the rigid k_x - ω relation gives way to a broader one, where a frequency can connect to a continuous k-range. The components whose phase velocities exceed the speed of light will radiate: that is linear bremsstrahlung.

The same is true when our wavefront hits a curved surface. Although "totally" reflected, one still should get synchrotron radiation at various angles. It becomes tangent only at v = c, when "traditional" total reflectivity ceases. It could be an issue for optical fibers.
We can understand this in terms of our Fourierdecomposition, this time for the angular coordinate of a cylindrical coordinate system. (with 1-d stripe)
For regular motion, we still get our rigid relation between frequency and angular wavevector

$$\omega(n) = v\frac{n}{R_0} \tag{16}$$

When we plug this into the wave equation, we get the ordinary Bessel equation whose solutions are Bessel and Neumann functions. They behave sinusoidally at greater distance.
Hence, we always get radiation.
When we switch to a narrow stripe (in z-direction), we get z-Fouriercomponents.
For given frequency, modes are radiative when $ck_z < \omega$ and become evanescent for $ck_z > \omega$.
If one wants maximum synchrotron radiation yield, one should use a 1-d stripe ($k_z = 0$)

Some observations

We notice that our radiative modes, for given frequency, are sensitive to small wavevectors, hence long wavelengths. Radiation becomes a collective phenomenon involving greater space.
If we had a "point-like" particle whose path has a sudden kink (collision), one immediately thinks of the emission of extremely short-wavelength radiation.
Sure, we need high frequencies to get the sudden kink but we need the space around that kink to produce the "superluminal" emissive situation.
To us who only see "some" present, this seems like hindsight when in the (for us) invisible reality, these extended modes are always there.

This throws up another problem: long-wavelength radiation from microscopic sources.
Atomic radiation, often visible, is in the (sub) μ-meter range.
From what we just saw, such radiation requires sources of similar size.
How come that sub-nanometer sources like atoms can achieve this stunt ?
The answer is, of course, that a tiny object contains all spatial modes to some extent through which radiation becomes possible. But the smaller the emitter the less its emissivity for long wavelengths.
Hence, the emissive character of an atomic species will change drastically as its particle density goes up. And so will the refractive index and its dispersion.
This should be an issue for a variety of things, from lasers to the blue sky.
We are only on the safe side when frequencies are so high that the wavelength lies within the emitting structure's size (nuclei).